FINANCING REPEAT BORROWERS: DESIGNING CREDIBLE INCENTIVES FOR TODAY AND TOMORROW

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Abstract. We analyze relationship lending when borrower cash flows are not contractible and the costs of intermediation vary over time. Because lenders provide repayment incentives to borrowers through the continuation value of the lending relationship, borrowers will condition loan repayment on the likelihood of receiving loans in the future. In other words, the lender's expected future financing decisions affect the value of the underlying loans. Thus, since future lending decisions are dependent on future lending costs, beliefs about the intermediary's future liquidity become an important component of the borrower's repayment decision. Consequently, the possibility of high lending costs in the future weakens repayment incentives, which in turn, leads to an inefficient under-provision of credit by the lender. Moreover, as the likelihood of a prolonged period of high liquidity cost increases, the adverse effect of future liquidity constraints on today's lending decisions exacerbates. We discuss the application of our model to the case of microfinance.

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1. Introduction

In many forms of financial intermediation, such as microfinance and trade credit, a lender has little formal recourse or enforcement power to procure repayment. This friction is especially prevalent in developing countries with weaker institutions and fewer creditor protections. Therefore, to stop ex-post moral hazard, a lender must design the contract to ensure that repayment is incentive-compatible for the borrower. This paper investigates dynamic incentives as one of the tools used by lenders to partially overcome this friction. We propose a simple model of relational contracting—informal agreements sustained by the value of the future relationships (Baker et al. [2002])—where borrowers derive utility from the future value of lending relationships, which in turn, induces repayment incentives today. In other words, the long-run value from the bilateral relationship provides short-run repayment incentives.

We consider a world with two frictions: First, borrowers cannot commit to repay their loans. Second, lenders face shocks to the cost of intermediation that are uncorrelated with the borrower's repayment ability. We believe the latter to be consistent with the case of microfinance, where global credit markets are virtually uncorrelated with the profits of micro-enterprises, and with the case of trade credit, where costs faced by suppliers may not be directly related to the demand for final goods faced by purchasers.

We illustrate how changes in the future value of a relationship will affect today's actions and subsequently the utility of both participants. We describe how the lender structures incentives to maximize profits within the constraint of being unable to write binding long-term contracts. We show that such incentive compatibility constraints, combined with changing intimidation costs, can lead to reduced lending and may even force the lender to forgo lending even when it is profitable to do so in the long-term. Furthermore, we detail how the lack of enforceable contracts can change the distribution of economic rents shared between the two parties. In particular, with a monopolistic lender, the borrower's utility may increase as she becomes more impatient because the future value of the relationship falls, decreasing the lender's capacity to charge higher interest rates.

We begin with a basic model of repeated lending where the borrower cannot commit to repay loans in any future state. Due to moral hazard, no lending is possible without the prospect of future trading. In the case where the lender's cost of lending does not change over time, we solve for the lender's optimal stationary relational contract and characterize the equilibrium. We then consider the lending environment when the lender's costs vary over time (according to a given Markov process). This added friction causes the lender to have the temptation to not offer loans in some periods due to the higher realized costs in those periods. Thus, we consider the ramifications of both the borrower and the lender being unable to commit to repaying or offering loans in all periods. We investigate equilibria where lending is possible in all periods and also equilibria where the lender may choose not to give loans in some periods. We show that in the latter case, partial default occurs along the equilibrium path solely due to the lender being unwilling to provide loans in some periods.

After solving the equilibrium of the game, we detail how the equilibrium changes as we alter the game's parameters. For instance, more persistent cost environments lead to a greater likelihood of the relationship breaking down (even when holding the ex-ante probability of each state constant). Therefore, this would support the idea that infrequent long recessions may be worse for relational contracting than frequent short recessions.

This paper is related to several strands of the economics and finance literature. First, it calls upon models of banking and intermediation under ex post moral hazard. Many related papers such as Townsend [1979] and Bernanke and Gertler [1989] show that under informational frictions, debt is the optimal type of financing by outside, uninformed investors. Bolton and Scharfstein [1996] consider the financing decision of the firm and the optimal debt structure when control can be transferred in different states of the world. In our setting, we also consider the characteristics of optimal financing, but we assume that borrowers have no net worth, that banks cannot improve the contracting environment through monitoring, and that the borrowers cannot transfer control of their projects to creditors.

Furthermore, there is the large financial literature (starting from Rajan [1992], Petersen and Rajan [1994], Berger and Udell [1995], Boot and Thakor [2000]) that studies how strong borrower-lender relationships can help to solve informational asymmetries. We contribute to this literature by modelling a lending relationship that is only possible through relationship lending—specifically, without the potential for a future lending relationship, the borrower has no incentive to repay the lender.

In this paper, we consider a lender that faces liquidity risk in the capital markets. In some periods, financing costs for the intermediary are high, while in other states, these financing costs are low. The liquidity channel and the potential benefits of securitization have been studied by Carlstrom and Samolyk [1995] and Parlour and Plantin [2008]. Drucker and Puri [2009] consider the potential diversification benefits of loan sales by financial intermediaries and show that borrowers whose loans are sold appear to receive more future credit from the same originator, potentially increasing the value of the lending relationship. In line with the results of our model, Berlin and Mester [1999] empirically show relationships between the structure of bank liabilities and relationship lending. Specifically, banks with greater deposit funding (relative to wholesale funding) provide greater loan smoothing than other banks.

Our paper also builds on the relational contracting literature (see Levin [2003], Halac [2012], and McAdams [2011], Macchiavello and Morjaria [2012], for example), which studies repeated contracting and reputation-building between parties. Our paper allows for transferable utility but assumes borrowers are credit constrained, preventing the transfer of decision rights to the agent who has the lowest reneging temptation.

1.1. **Motivating Examples.** While our framework applies to any setting where contract enforcement between two parties today depends on the future value of the relationship between them, we highlight two applications—microfinance and trade credit.

Microfinance. Microfinance has been a rapidly growing and well-publicized financial tool in developing countries.[1] Microloans are intended for poor clients who generally do not have any pledgeable collateral and who live in countries where the formal legal institutions have little to no ability to enforce small credit contracts. To overcome these constraints, microlenders have developed two important contractual innovations to make lending feasible: social collateral and dynamic incentives. Microfinance is able to harness the power of social networks in a variety of

[1]According to mixmarket.org, microfinance institutions disbursed more than $150 billion in loans to over 30 million customers worldwide in September 2018.

ways to provide repayment incentives.[2] However, in this paper we focus on the use of repeated contracting to provide repayment incentives. The standard microloan tends to be worth approximately $100 and is generally meant to be used to finance the purchase of assets or working capital for microenterprises. Borrowers make frequent installment payments for one year and upon completion, are disbursed a new loan.[3] If borrowers default on their loans, then they are not offered future access to credit. This dynamic incentive is often credited as one of the main innovations of microfinance.[4]

In connection with our model, we highlight the potential fragility of microfinance institutions (MFIs) as a result of the reliance on future lending activities to provide repayment incentives today. Namely, if microlenders themselves become liquidity-constrained in the future, does this constraint have feedback effects on the repayment of existing, outstanding loans? In this type of scenario, common financing structures such as short-term debt or loan securitization may be suboptimal. This analysis is relevant to recent developments in microfinance in India, one of the largest microfinance markets, that has faced similar crisis in the past.[5] Throughout the crisis, microlenders faced difficulties accessing credit markets, despite the fact that the borrowers' economic conditions remained unchanged. Some smaller MFIs failed to obtain financing and walked away from their loan portfolios. Larger MFIs were forced to delay new disbursements while still paying staff costs and assuring clients that their new loans were indeed coming.

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[2]Many microfinance contracts have joint liability, where borrowers must pay for their peers if those peers decide to default (see Gine and Karlan [2006], Gine and Karlan [2009], and Giné et al. [2011] for empirical evidence on the effects of joint liability.) Research has also shown (see Feigenberg et al. [2010] and Breza [2012]) that even without contractual linkages between borrowers, there still may be social effects at play.

[3]In many settings the standard maturity for a microloan is 50 weeks, but some lenders offer variations on this product.

[4]See Morduch [1999]. Using an innovative empirical design, Karlan and Zinman [2009] find that borrowers do respond to dynamic incentives and are more likely to repay loans if they know a new loan is forthcoming.

[5]In October 2010, the Indian state of Andhra Pradesh suffered from a debilitating wave of microfinance defaults sending repayment rates from 100% to 10% and below. The default crisis began when the state government passed an emergency ordinance to rein in microfinance institutions. The ordinance set out to curb usurious interest rates, cases of borrower harassment by credit officers and over-borrowing by the state's poor. Stories of borrower suicides permeated the local and international press (see for example don [2010] and IBT [2010]). The localized crisis spread to all of India through a national liquidity crisis, when banks refused to lend to MFIs.

To apply our model to the case of microfinance, we make two financial friction assumptions. First, we assume that MFIs are monopolist lenders. We argue that this assumption is innocuous because some sort of monopoly power is required to maintain repeated lending relationships. Furthermore in the case of India, regulation gives MFIs some degree of market power. There, borrowers are allowed to take loans from a maximum of two lenders.[6] Second, in our model, any frictions would be minimized if lenders could be financed completely by equity. We assume, however, that MFIs must access the debt markets in order to grow. This assumption can be justified by assuming either that it is impossible to transfer control to debt holders or that equity-holders of intermediaries must be informed, and informed capital is scarce.[7]

Trade credit and supplier relationships. In the case of trade credit and supplier relationships, McMillan and Woodruff [1999] and Banerjee and Duflo [2000] analyze the value of reputation and repeated interactions. However, the prevailing literature such as Burkart and Ellingsen [2004] and Cunat [2007] analyze the decisions of firms to offer trade credit but do not consider how a borrower's incentive to repay the trade credit would depend on changes in the supplier's capacity to either offer more goods, or continue to finance the stock or services. This concept of supplier trade credit would be most important in the developing world where contractual protection is weaker.[8]

This paper explicitly examines a buyer's (borrower's) incentive to renege on repaying the supplier (lender) as a function of the supplier's capacity to continue to deliver future products (loans). Even though, our model description is of lender-borrower relationships, a relabeling of repayment to be relationship specific investment allows us to consider how investment by both parties changes over time as the cost of investing changes.

In the next section, we describe our environment. Section (3) presents the baseline case where the lender's cost does not change over time. Section (4) extends the

[6]To protect borrowers against price gauging by monopolists, interest rates are also regulated at a maximum rate of 10%-12% above the lender's cost of capital.
[7]The requirement that intermediaries be informed can be found for example in Holmstrom and Tirole [1997] and the extension to microfinance by Conning and Morduch [2011].
[8]In the US, supplier trade credit is usually recoverable even if the supplier goes bankrupt, however this is not the case worldwide.

analysis to our full model by allowing for the cost of lending to vary over time. Section (5) concludes.

2. Model

This section describes the model, assumptions, timing, and solution concept. Using this model, Section (3) characterizes the solution for a basic version of this model that has no change in the lender's funding cost and Section (4) extends to this model by allowing the lender's funding cost to follow a Markov process.

2.1. **Set up.** There is an infinitely repeated game between a risk-neutral, profit-maximizing, monopolist lender and a risk-neutral capital-constrained agent.

Each period, the agent has an investment project that requires a unit of capital and produces a deterministic output of v at the end of the period. The lender can provide a loan to the agent for the project. The lender's cost of a unit of capital is c_t, which takes two possible values, c_l and c_h, where c_l is strictly less than c_h.[9] Upon lending, the lender requests a repayment of r_t to be made at the end of the period.[10]

When the agent invests in their project, the output is observable and verifiable but, crucially, the lender does not have any enforcement power. Rather, the bank's sole tool to extract repayment for the loan is through a relational contract and ensuring the value of a long-term banking relationship outweighs the short-term incentive to renege on payment.

To highlight the importance of the future relationship between the lender and the agent, we assume that the lender's cost of capital for the next period, c_{t+1}, is publically observable prior to the agent repaying the lender. Therefore, before the agent decides whether to repay, the agent can infer whether the lender will offer a future loan, and if so, at what interest rate. This assumption is motivated by two different phenomena: (i) microfinance lenders have borrowers on different loan cycles, therefore, if a borrower's friend is unable to renew their loan, this information will influence the borrower's expectation of whether they will get a loan tomorrow and

[9] c_t can be thought of as a reduced form formalization of the cost of lending in each period. In particular: $c_t \equiv \kappa_e c_{e,t} + (1-\kappa_e)c_{d,t}$ such that κ is between 0 and 1. $c_{e,t}$ is the cost of equity financing, $c_{d,t}$ is the cost of debt financing, and κ is the bank's capital to assets ratio.
[10] r_t includes both the loan's principal and interest payment.

(ii) the information concerning microfinance institutions is pretty well documented in both newspapers and by politicians.[11]

The evolution of lending cost (c_t) follows a Markov process whose transition matrix is common knowledge:

$$\begin{bmatrix} p_l & 1-p_l \\ 1-p_h & p_h \end{bmatrix}$$

where $p_l = \text{Prob}(c_{t+1} = c_l | c_t = c_l)$ is the probability of moving from a low-cost state to a low-cost state and $p_h = \text{Prob}(c_{t+1} = c_h | c_t = c_h)$ is the probability of moving from a high-cost state to a high-cost state.[12] Thus, the long-term stationary (invariant) distribution of states is:

$$\Theta = \begin{bmatrix} \theta_l \\ \theta_h \end{bmatrix} = \begin{bmatrix} \frac{1-p_h}{2-p_l-p_h} \\ \frac{1-p_l}{2-p_l-p_h} \end{bmatrix} \tag{1}$$

where θ_l is the long-run probability of low cost and θ_h is the long-run probability of high cost.

For ease of exposition, we make a number of simplifying assumptions. The agent's and the lender's outside options are both normalized to zero and the borrower is credit-constrained such that they cannot pay any amount ahead of time. The agent also has no storage technology to transfer wealth across time and we assume that the agent and the lender have the same common discount rate, $\delta \in (0, 1)$.

While the bank and the agent cannot enter in any form of binding contract, before the game begins, the lender offers a "relational contract" that lays out the conditions under which the lender will lend the required capital to the agent and the required repayment amount in any given time period.[13] The relational contract also specifies the action undertaken by the lender or the borrower if the other party fails to undertake their specified action in the relational contract.

Once the lender offers the relational contract, if the borrower accepts the contract, the repeated game begins. The lender's goal is to choose a relational contract that maximizes their expected discounted stationary profits in the repeated game,

[11]For example, during the recent Indian microfinance crises, newspapers and politicians have written damning articles about MFIs and bankrolled television commercials to further undermine the portfolios of MFIs after the onset of the crisis. See Banerjee and Duflo [2011] and Breza [2012] for discussions of the recent Indian microfinance crises.

[12]The probability of moving from a low state to a high state is therefore $(1 - p_l)$ and the probability of transitioning from a high state to a low state is $(1 - p_h)$.

[13]This can entail lending in every period or just some periods.

conditional on being accepted by the borrower and being incentive-compatible at all times.

We solve for the perfect public equilibrium of this game which requires that play following each history be a Nash equilibrium.[14] Moreover, for simplicity and ease of exposition, we focus solely on stationary relational contracts where loan schedule and repayment amounts are independent of time.

2.2. **Timing.** The timing of the game is as follows:

(1) The lender offers a relational contract to the potential borrower.
(2) The borrower decides weather or not to accept the relational contract. If the borrower rejects the contract, the game ends and the lender and the agent both receive zero payoffs. Otherwise, the game proceeds.
(3) Both parties observe the lending cost in the first time period (c_1) and the repeated game begins.
(4) At the beginning of each time period t (including $t = 1$):
    (a) The lender chooses whether to offer a loan. Their decision is denoted by $l_t \in \{0, 1\}$ with $l_t = 1$ for lending. If the lender offers a loan, the lender also chooses a required repayment amount r_t.
    (b) The agent either accepts or rejects the loan offer. Their decision is denoted by $d_t \in \{0, 1\}$ with $d_t = 1$ for accepting.
(5) At the end of each time period t (including $t = 1$):
    (a) The lending cost for the next period (c_{t+1}) is observed by all parties.
    (b) The borrower then decides how much to repay $\hat{r}_t \in [0, r_t]$.
    (c) Payoffs are realized.
(6) The game enters period $t + 1$ and repeats from step (4).

[14]In this model, perfect public equilibrium imposes the same sequential rationality requirement that subgame perfection would impose in a complete information model [Fudenberg et al., 1994]. Hence, once the repeated game starts, neither the lender nor the borrower can commit to abiding by the relational contract and each will unilaterally withdraw from the agreement in any time period if they do not find it in their best interest to continue. In other words, the bank's choices are limited to relational contracts that are incentive-compatible, that is, prefect public equilibria. Simply put, the pre-game step works as an equilibrium refinement criteria for cases where the repeated game has more than one equilibrium. Such setting is consistent with the reality of micro-finance where formal contracting is not possible but the bank can implement dynamic repayment incentives by putting preconditions on subsequent loans.

We first analyze a benchmark case where lending cost is constant in Section (3). Then we turn our attention to finding the equilibrium of the full model in Section (4).

3. Equilibrium: Relational contracts with no change in cost over time (baseline)

To gain intuition into the lender and borrower's dynamic incentives, we begin with a simple environment that excludes any uncertainty. Specifically, we start with a constant cost of lending in all time periods, that is, $c_t = c$ in all time periods.

By abstracting from the time-varying cost structure in the full model, we consider a simple stationary version of our problem to demonstrate what inefficiencies in lending may arise due to the absence of binding contracts.

In the simple baseline model, the lender chooses a feasible relational contract that maximizes their total discounted profits:

$$\Pi = \sum_{t=0}^{\infty} \delta^t l_t (r_t - c) \tag{2}$$

Recall that the relational contract is not a binding contract and thus has to be sustainable in equilibrium, that is, it must be both incentive-compatible for the lender and also satisfy the borrower's repayment incentive compatibility constraint at every point in time. In other words, the borrower must be willing to make the repayment the bank requires in every period, that is, $\hat{r}_t = r_t$. To ensure the latter, using Abreu [1988] one-shot deviation principle, we know that the optimal lender punishment for non-compliance is breaking off trade for all future rounds. Therefore, the borrower's long-term payoff of complying at the end of each period must be greater than or equal to the borrower's outside option:

$$\sum_{t=1}^{\infty} \delta^t l_t (v - r_t) - r_0 \geq 0 \tag{3}$$

where $\sum_{t=1}^{\infty} \delta^t l_t (v - r_t)$ is the discounted value of future borrower surplus from borrowing, and r_0 is the required repayment at time $t = 0$. Thus, the lender's constrained maximization problem is as follows:

$$\max_{\{l_t, r_t\}_{t=0}^{\infty}} \Pi \text{ (as defined by equation [2])}$$
$$\text{s.t. equation [3] holds.} \tag{4}$$

Furthermore, recall that we concentrate solely on stationary relational contracts where contract parameters are time-invariant, that is, $l_t = l$ and $r_t = r$ for all t. Therefore, there are only two possible types of relational contracts: no lending in any period ($l = 0$) or the following:

i. The lender extends a loan and requires repayment r in the first period.
ii. After the first period, at all histories where the lender has always received a repayment of at least r in the past, the lender will continue to extend a loan and require repayment r. At any other history, the lender will not offer any loans.
iv. The borrower accepts a loan and repays r in the first period.
v. After the first period, at all histories where a loan has been extended in all previous periods, the borrower accepts a loan and repays r. At any other history, the borrower accepts a loan and repays zero.

The relational contract described above, will be a perfect public equilibrium so long as both parties' long-run discounted payoff is non-negative. However, since we assume that the lender makes the initial offer, the lender opts for the equilibrium that generates the largest profits for the them, which is the equilibrium with the highest repayment r^* that is acceptable to the borrower. The result would be the solution to the lender's maximization problem laid out in equation [4].

To pin down r^*, we begin with noting that in a stationary relational contract, the borrower's ICC (equation [3]) simplifies to:

$$\frac{\delta}{1-\delta}(v - r) - r \geq 0 \tag{5}$$

The left hand side of equation [5] is the value of sustaining the relationship and the right hand side is the borrower's outside option. The latter is the result of the fact that as discussed earlier, the optimal punishment for reneging is breaking the relational contract so that the game reverts to the static Nash equilibrium actions, in which no loans are offered.

Solving equation [5] for r reveals that the maximum repayment r^* that can be sustained in a stationary equilibrium is δv.

Turning to the lender's problem of choosing the most profitable stationary relational contract, equation [4] simplifies to maximizing $r - c$ conditional on the expected profits in the sustained relationship being higher than the outside option of zero. Therefore, the profit-maximizing monopolist lender will lend, that is, $l^* = 1$ and require a repayment of $r^* = \delta v$ in every period if $\delta v > c$. Otherwise, no stationary relational contract with positive lending is possible because even with maximal punishment (no future loans), the borrower is unwilling to make a repayment that is high enough for the bank to cover its costs.

Proposition 1. *When intermediation costs are fixed over time at c, lending is possible if, and only if, $c \leq \delta v$. Under this condition, the bank will lend ($l^* = 1$) and require a repayment of $r^* = \delta v$ in every time period. Otherwise, no lending occurs in equilibrium.*

Proposition 1 states that even though the lender is a monopolist, it cannot expropriate the full rent from the relationship by charging $r = v$ due to the lack of formal contractual protection. In fact, the only obstacle that prevents the borrower from opportunistically reneging is her "shadow value of the future" that is equal to δv. The latter is only sufficient to constrain the borrower from reneging if the repayment does not exceed the shadow value of the future. Thus, the borrower retains a positive per-period payoff equal to $(1 - \delta)v$. Moreover, lending does not occur at costs above δv even though it is socially efficient to lend so long as $c < v$.

4. Equilibrium: Relational contracts with changing costs

In this section we turn our attention to the full model where the cost parameter changes over time. In particular, we let the lenders cost of lending c_t, take two possible values, c_l and c_h, such that c_l is less than δv and c_h is greater than δv (that is, $c_l < \delta v < c_h$).[15]

[15] In the other two possible cases, the equilibrium characterization is trivial. If cost is always below the borrower's shadow value of the future, that is, $c_l < c_h < \delta v$, then lending in every period is profitable for the bank and the lending equilibrium characterized in Section (3) continues to hold. If cost is always more than the borrower's shadow value of the future, that is, $\delta v < c_l < c_h$, then lending is never profitable and no lending occurs in equilibrium.

Recall the assumption that at the end of each time period t, the borrower observes tomorrow's cost realization c_{t+1} *prior* to deciding whether to repay today's loan. This assumption, combined with the variability of lending costs, captures the possibility that the borrower's moral hazard—the borrower's incentive to renege—changes over time.

As before, the lender and the borrower cannot enter into a binding contract. Nonetheless, they can agree on a relational contract (l_t, r_t) if it is incentive-compatible for both the lender and the borrower. The borrower's incentive compatibility constraint is:

$$\forall t \text{ s.t. } l_t = 1, \quad \sum_{k=1}^{\infty} \delta^k E[l_{t+k}(v - r_{t+k})] - r_t \geq 0 \tag{6}$$

which requires the borrower's expected discounted payoff of complying to be not smaller than their outside option at time t.[16] Moreover, the lender should also find it profitable to abide by the relational contract at any point in the game, that is,

$$\forall t \quad \Pi_t = \sum_{k=0}^{\infty} \delta^k E[l_{t+k}(r_{t+k} - c_{t+k})] \geq 0 \tag{7}$$

At the beginning of the game, the lender's goal is to choose a relational contract that maximizes the lender's long-term expected stationary profits, conditional on that both ICCs, that is, equations [6] and [7], are satisfied.

$$\begin{aligned} &\max_{\{l_t, r_t\}_{t=0}^{\infty}} \Pi = E_\Theta(\Pi_0) \\ &\text{s.t. equations [6] and [7] hold.} \end{aligned} \tag{8}$$

As before, we limit our attention to stationary relational contracts, where lending and repayment are independent of time but can be state-dependent.

Definition 1. A relational contract (l_t, r_t) is stationary if for all t:

$$l_t = \begin{cases} l_h \text{ if } c_t = c_h \\ l_l \text{ if } c_t = c_l \end{cases} \quad \& \; r_t = \begin{cases} r_h \text{ if } c_{t+1} = c_h \\ r_l \text{ if } c_{t+1} = c_l \end{cases}.$$

Stated differently, the lender can offer a loan and promise to continue lending in both or one of the states as long as the borrower repays the agreed amount. Such

[16]By Abreu [1988], the optimal punishment for non-compliance is breaking off trade for all future periods.

an offer would be accepted by the borrower and respected by both parties only if it satisfies both the borrower and the lender's incentive compatibility constraints (that is, equations [6] and [7])—the relational contract's feasibility conditions. Then, conditional on the relational contract being feasible, the lender offers the relational contract that generates the largest expected profit for the lender. There are three possible types of stationary relational contracts that the lender can choose from:

(1) **Always-lending**—offer to lend in all states.
(2) **Partial-lending**—offer to lend is state-dependent.
(3) **No-lending**—the lender does not offer any loans.

The no-lending relational contract's feasibility and equilibrium characterization are trivial. Once the conditions under which each of the other two relational contracts are feasible are found, and their respective equilibria characterized, the lender's optimal choice can be established by comparing the long-term stationary payoffs of each relational contract. The lender will choose the relational contract that produces the highest expected profits.

In Proposition (2), we state the lender's optimal relational contract and subsequently explain the intuition for the lender's preferred strategy and the game's characterization. Then in Sections (4.1) and (4.2), we explain in detail the lender's and borrower's strategies for each relational contract.

Proposition 2. *At the beginning of the game:*

(Always-lending relational contract) *The lender offers to lend in all states if, and only if, conditions (1) and (2) hold:*

(1) $c_h - \delta v \leq \delta \left(\frac{1-p_h}{1-\delta p_l}\right)(\delta v - c_l)$*, which guarantees always-lending to be feasible.*
(2) $\theta_h c_h \leq \delta v - \delta v \theta_l \left(p_l + (1-p_l)\frac{\delta(1-p_h)}{1-\delta p_h}\right)$*, which guarantees always-lending to be more profitable than partial-lending.*

(Partial-lending relational contract) *Alternatively, the lender offers to lend only in the low-cost state if, and only if, condition (2) does not hold and the following condition holds:*

(3) $c_l \leq \delta v[p_l + (1-p_l)\frac{\delta(1-p_h)}{1-\delta p_h}]$*, which guarantees partial-lending to be feasible.*

(No-lending relational contract) *Otherwise, if neither one of conditions (1) and (3) hold, then lending is not feasible and the lender opts out of lending.*

The proof of Proposition (2) is in the Appendix.

Proposition (2)'s conditions (1) and (3) are the feasibility conditions for the always-lending relational contract and the partial-lending relational contract. Condition (2) is the lender's profit maximising condition, and if it holds, the always-lending relational contract is more profitable than partial-lending relational contract in expectation. Conversely, if condition (2) does not hold, partial lending is more profitable than always-lending. Finally, if the always-lending or partial lending relational contracts are both infeasible, then the lender makes no loans (no-lending).

To build intuition for Proposition 2, figure (1) shows how the feasible set of equilibria changes as we adjust the persistence of the high-cost state (p_h) on the x-axis, the cost of lending in the high-cost state (c_h) on the y-axis, and keeping all other parameters constant. As expected, if the persistence of the high state is high and the cost of lending in the high state is high (the top-right of the figure), no-lending is feasible (the area denoted as "N" on the figure). In contrast, if p_h and c_h are both low (the bottom-left of the figure), both always-lending and partial lending are feasible equilibria (the area denoted as "B" on the figure"). As we assume the lender is able to choose their preferred equilibrium at the start of the game, the final equilibrium will be determined by whichever equilibrium provides the lender with the higher expected profits [Proposition (2)'s condition (2)].

If only one of p_h or c_h are high, we get contrasting equilibria. If c_h is high but p_h is low, only the partial-lending relational contract is feasible (the area denoted as "P" on the figure). Intuitively, as the cost of lending in the high state rises (c_h), the lender is less inclined to always-lend because the lender will make a larger loss that period (recall that the cost of lending in the high state is larger than the maximum repayment from the borrower), whereas, the feasible set of partial-lending relational contracts is unchanged as c_h rises because the lender never lends when the the lending cost is high. If c_h is low but p_h is high, only the always-lending relational contract is feasible (the area denoted as "A" on the figure). In this scenario, the lender's losses from lending in the high-cost state are relatively smaller and the high persistence of the high-cost state (p_h) limits the economic surplus from only partially lending.

Figure (1) shows the feasible set of relational contracts whereas figure (2) shows the lender's preferred relational contract (the contract that confers the highest expected discounted profits for the lender) given the set of feasible relational contracts.

FIGURE 1. Feasible equilibria for different high-cost and persistence parameters

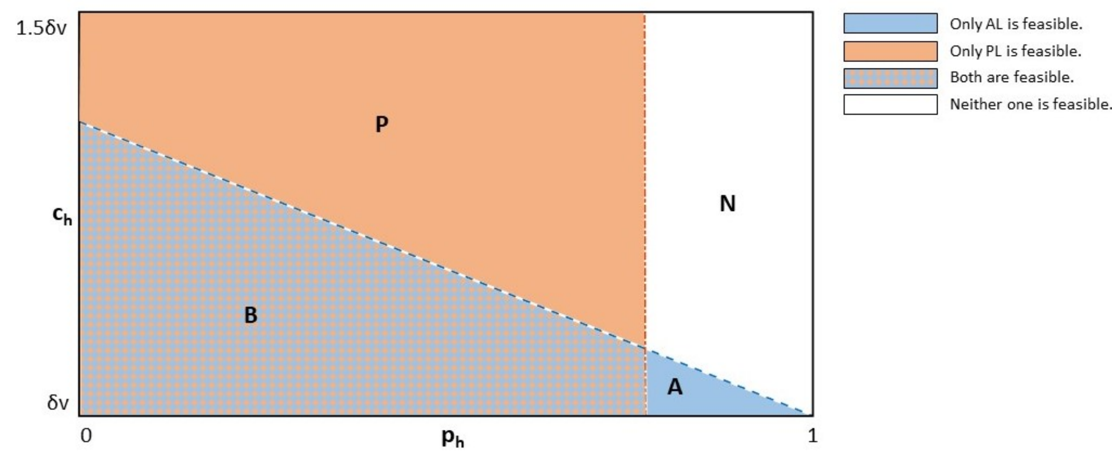

Figure (1) shows how the feasible set of equilibria changes as we adjust the persistence of high states (p_h) on the x-axis and the cost of lending in the high-cost state (c_h) on the y-axis and keeping all other parameters constant.

FIGURE 2. Lender's preferred equilibrium for different high-cost and persistence parameters

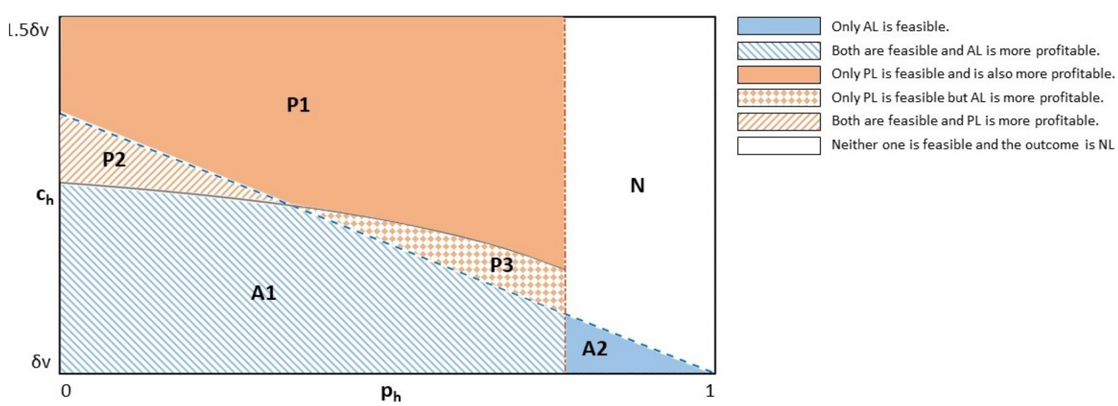

Figure (2) shows how the lender's preferred relational contract changes as we adjust the persistence of high states (p_h) on the x-axis and the cost of lending in the high-cost state (c_h) on the y-axis and keeping all other parameters constant.

Figure (2) shows three interesting areas: "P2", "A1", and "P3". The areas denoted as "P2" and "A1" show the set of parameters where both partial-lending and always-lending are feasible. However, the lender's preferred relational contract is partial-lending for the set of parameters in area "P2" and always-lending for the set of parameters in area "A1". Intuitively, as the cost of lending in the high state (c_h)

rises, partial-lending becomes relatively more profitable (recall, in partial-lending, the lender does not make any loans in the high-state).

The area "P3" shows the partial-lending relational contract as the lender's preferred strategy, despite the fact that if the always-lending relational contract was feasible, this contract would have generated *greater* discounted expected lender profits than the partial-lending relational contract. This result follows from the assumption that the lender cannot commit to lend in all states. Therefore, when a high-cost state is realized, for parameters in the "P3" area, the lender's temptation to stop lending is greater than the expected benefits from keeping to lend (that is, equation [7] is not satisfied for the always-lending relational contract when the state is high). Consequently, the always-lending relational contract is not feasible, even though, if the lender could commit to lend in all states from the start of the game, this contract would generate higher expected profits for the lender.

To formally prove Proposition (2), we characterize the feasibility conditions and the lender's profits for each of the three possible strategies, and subsequently derive the lender's optimal strategy, and consequently the game's equilibrium for different parameters. Sections (4.1) and (4.2) characterize the always-lending and the partial-lending relational contracts respectively.

4.1. **Always-lending: The lender offers a loan in all states.** If the borrower believes that the lender will offer a loan in every period, then the borrower's participation decision problem will be identical to the case where the lender's cost is deterministic, as discussed in Section (3). Thus, as we show in section (3), the borrower's maximum incentive-compatible repayment is δv for all time periods. Then using the borrower's maximum willingness to repay and the lender's expected profit (as shown in equation [7]) reveals that by extracting a maximum repayment of $r_t = \delta v$ in every period, the lender's expected discounted profit at time $t = 0$ simplifies to:

$$\Pi_0 = E\left[\sum_{t=0}^{\infty} \delta^t (\delta v - c_t) | c_0\right] \tag{9}$$

To ensure offering loans in every period is a credible strategy, equation [9] must be greater than zero for both $c_0 = c_l$ and $c_0 = c_h$, otherwise the lender could not credibly offer loans in all states. This equation can be written in recursive form by designating the lender's value function for each state as V_s where s is the current state, and s' as the state in the next period. Then, for the lender to not default the following ICC must hold:

$$V_s = (\delta v - c_s) + \delta E(V_{s'}|s) \geq 0 \quad \forall s = \{l, h\}$$

The above ICC requires the current value from being in the relationship be sufficiently large such that the lender would prefer not to walk away.

Lemma 1.

a) The lender will only be able to credibly offer loans in all states if the following condition is satisfied:

$$(1 - \delta p_l)(\delta v - c_h) + \delta(1 - p_h)(\delta v - c_l) \geq 0$$

b) For a given stationary distribution of the Markov process (Θ), the left hand side of the above inequality is decreasing in the probability of persistence of the high-cost state, that is, p_h.

The proof of lemma (1) is in the Appendix.

The intuition for Lemma (1b) is that the lender makes a loss in every period that the cost is high, therefore, as we increase the probability of staying in a high-cost state, the lender's incentive to give loans falls. This result suggests that the expectation of prolonged periods of high costs for the lender makes lending in all states more difficult, even when the ex-ante long-term likelihoods of being in the low-cost and the high-cost states do not change. In particular, the stationary (state-independent) expected profit of the lender, as defined by equation [8], can be written as:

$$\Pi = E_\Theta(\Pi_0) = \sum_{t=0}^{\infty} \delta^t \left[\theta_l(\delta v - c_l) + \theta_h(\delta v - c_h)\right] \tag{10}$$

By Lemma (1) and equation [10], for a given stationary distribution of the Markov process (Θ), while the long-term expected profit of the lender is constant, the always-lending relational contract becomes harder to maintain as the probability of a prolonged bad state increases.

Turning to efficiency, in order for the lender's stationary expected profit to be positive (that is, $\Pi \geq 0$), by equation [10], the following inequality must hold: $\theta_l(\delta v - c_l) + \theta_h(\delta v - c_h) > 0$. Plugging in for Θ from equation [1], we get:

$$(1 - p_h)(\delta v - c_l) + (1 - p_l)(\delta v - c_h) > 0$$

Comparing the left hand side of the above inequality to that of Lemma (1a) reveals that the latter is smaller for a given set of parameters. Thus, even if the above inequality holds and an always-lending relational contract is profitable for the lender at the beginning of the game, it may not be time consistent for the lender. Specifically, the lender's renege temptation—that is to not lend—is greater when the lender has high costs, whereas, the lender's ex-ante profitability constraint is before the state of the world is observed, which by definition must be easier to satisfy. To sum, lenders with very high costs in some periods, will not be able to lend in all periods even when it is long-term profitable to do so because the lender cannot credibly commit to lend when a high-cost state is realized.

This last result is in contrast to the baseline case that we analyzed in Section (3) where the lender always participates so long as always-lending is ex-ante profitable. Thus, uncertainty introduces an additional layer of inefficiency. Nonetheless, in the stationary relational contract explored in this section, since lending in all states is possible, there are no inefficiencies in the provision of credit. In the next section, we explore the additional inefficiencies that arise when costs in the high-cost state are too high so that the lender is not able to offer loans in all periods.

4.2. **Partial-lending: Lender only offers a loan in the low-cost state.** The lender's cost of lending may be prohibitively high in some periods causing the lender to only offer loans in the low-cost state. As such, the borrower's incentives to continue repaying loans will diminish due to the lower likelihood of receiving future loans. Then, the lender has to accept partial repayments in cases where the cost in the next period is high. Specifically, the lender sets an interest rate r_l for all loans, which the borrower has to pay if the state in $t+1$ is c_l or lose access to all future loans. However, when the state in $t+1$ is c_h, the borrower is allowed to partially default and only repay $r_h < r_l$. In the latter case, if the borrower repays less than r_h then the lender refuses to offer any future loans.[17] This partial-lending relational contract, can formally be represented with (r_l, r_h) such that:

(1) the lender's side of the relational contract is described as:
   i. Offer a loan in the first period if, and only if, cost is c_l.

[17]We do not rule out situations where $r_h = 0$.

ii. At all histories where the lender has offered a loan whenever cost was c_l and received at least r_s when the next period's state was s, continue to offer a loan whenever cost is c_l.

iii. At any other history, do not offer a loan.

(2) the borrower's side of the relational contract is described as:

i. Accept a loan and repay $r_{s'}$ in the first period if a loan is offered, where s' is the next period's state.

ii. At all histories where the lender has offered a loan whenever cost was c_l, accept a loan and repay $r_{s'}$, where s' is the next period's state.

iii. At any other history, accept a loan and repay zero.

In this case, the lender's expected discounted profit at time $t = 0$ simplifies to:

$$\Pi_0 = E\left[\sum_{t=0}^{\infty} \delta^t l_t(r_t - c_t)|c_0\right] \ \forall c_0 \in \{c_l, c_h\} \tag{11}$$

and the borrower's incentive compatibility requires:

$$E\left[\sum_{t=1}^{\infty} \delta^t l_t(v - r_t)|c_1\right] \geq r_0(c_1) \ \forall c_1 \in \{c_l, c_h\} \tag{12}$$

where $r_0(c_s) = r_s$.

Equation [12] can be rewritten in the following recursive forms where the subscripts denote today's state and tomorrow's state respectively:

$$\begin{aligned}
U_{LL} &= \quad v - r_l + \delta\left[p_l U_{LL} + (1-p_l)U_{LH}\right] && \geq v \\
U_{LH} &= \quad v - r_h + \delta\left[p_h U_{HH} + (1-p_h)U_{HL}\right] && \geq v \\
U_{HH} &= \quad 0 + \delta\left[p_h U_{HH} + (1-p_h)U_{HL}\right] && \geq 0 \\
U_{HL} &= \quad 0 + \delta\left[p_l U_{LL} + (1-p_l)U_{LH}\right] && \geq 0
\end{aligned} \tag{13}$$

Since U_{HL} is a weighted average of U_{LL} and U_{LH}, and U_{HH} is a fraction of U_{HL}, the last two inequalities are non-binding. Intuitively, the borrower has more incentive to renege when the current state is low and she has already received a loan than when she has not received a loan. Thus, we restrict our attention to the first two constraints in equation [13] that are potentially binding. They can be rewritten as:

$$\begin{aligned}
v\delta(1-\delta p_h) &\geq [(1-\delta p_h) - \delta^2(1-p_l)(1-p_h)]r_l + \delta(1-\delta p_h)(1-p_l)r_h \\
v\delta^2(1-p_h) &\geq \delta^2 p_l(1-p_h)r_l + (1-\delta p_h)(1-\delta p_l)r_h
\end{aligned} \tag{14}$$

Since, in a partial-lending relational contract, loans are only disbursed in the low-cost state, the lender's problem, that is to maximize Π_0 as described in equation [11], can be simplified to maximizing the expected per-period profit in the low cost state:

$$\begin{aligned} \max_{r_l, r_h} \quad & p_l r_l + (1-p_l) r_h - c_l \\ & \text{s.t. equation [14] holds} \\ & \text{and } r_l \geq r_h \geq 0 \end{aligned} \tag{15}$$

The resulting optimization problem can be solved with a linear program.

Lemma 2.

a) The lender will only be able to credibly offer loans only in the low-cost state, if, and only if, the following condition is satisfied:

$$\delta v[p_l + \frac{\delta(1-p_l)(1-p_h)}{1-\delta p_h}] \geq c_l$$

b) In this relational contract, the lender chooses the following requisite interest payments:

$$\begin{aligned} r_l^* &= \delta v \\ r_h^* &= \delta v \frac{\delta(1-p_h)}{(1-\delta p_h)} \leq r_l^* \end{aligned}$$

The proof of Lemma (2) is in the Appendix.

Lemma (2a) establishes the lender's incentive compatibility constraint for the profit-maximizing partial-lending relational contract.

In contrast to the always-lending equilibrium, offering loans only in the low-cost state becomes *easier* when states are more likely to persist. To see this, the inequality in Lemma (2a) reveals that, holding the stationary distribution of the Markov process (Θ) constant, as the probability of persistence of the high-cost state (p_h) converges to 1, the left hand side of the lender's ICC converges to δv.[18] Since δv is greater than the cost of lending in the low state, c_l, the lender's ICC is satisfied in the limit. The intuition for this result follows from the fact that the lender only offers loans in the

[18]Recall that varying p_h while holding Θ constant requires $p_l = \frac{2\theta_l + p_h\theta_h - 1}{\theta_l}$. Thus, $\lim_{p_h=1} p_l = 1$. As a result $\lim_{p_h=1} p_l + \frac{\delta(1-p_l)(1-p_h)}{1-\delta p_h} = 1$.

low state, therefore, the lender only receives lower payment when the state moves from low-cost to high-cost. That friction weakens as the state of the world becomes more persistent. Moreover, since the lender does not lend during a high-cost period, the persistence of high costs does not have an effect on her participation decision.

5. Discussion

The model describes how the future affects the borrower's decision today, and in turn affects the strategy undertaken by the lender. Furthermore, the potential to renege occurs for both parties: the borrower may renege on repaying the lender; the lender could renege on the implicit promise to offer loans. Thus, even if ex-ante, the lender's profits were maximized by offering loans in all states, this may not be credible due to the potential to renege when the cost of lending is high. The equilibrium of the game may stipulate states of the world where no loans are given and where full repayment is not expected.

One implication is that the lender may have difficulties financing its operations and its loan origination activities. Because of the nature of the equilbria described above, equity financing on the part of the lender would lessen these concerns because the long-run equity investor would be able to reap the benefits of offering loans at a loss in some periods. In many real world settings, however, relationship lenders frequently access the short term debt markets for financing. In this case, our model suggests that short term debt, and therefore the securitization of individual loans, may be a problematic financing structure for loans that rely on dynamic repayment incentives.

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Appendix

Proof. of Proposition 2

The stationary expected per-period profits when lending in all states is $\delta v - \theta_l c_l - \theta_h c_h$ and when lending only in the low-cost state is $\theta_l(p_l r_l^* + (1-p_l)r_h^* - c_l)$. Thus, after plugging in for r_l^* and r_h^*, the lender's expected profit is higher when lending in every period if, and only if,:

$$\delta v - \theta_l c_l - \theta_h c_h \geq \theta_l \left[\left(p_l + \frac{\delta(1-p_l)(1-p_h)}{1-\delta p_h} \right) \delta v - c_l \right]$$

which with some algebraic manipulation gives:

$$\delta v \left[1 - \theta_l \left(p_l + \frac{\delta(1-p_l)(1-p_h)}{1-\delta p_h} \right) \right] \geq (1-\theta_l) c_h$$

The remainder is based on lender credibility conditions (ICCs) given by Lemmas 1 and 2. □

Proof. of Lemma 1

Assuming the lender always offers loans we can write V as:

$$\begin{aligned} V_s &= (\delta v - c_s) + \delta[p_s V_s + (1-p_s)V_{-s}] \qquad \forall s \\ &= \frac{(\delta v - c_s) + \delta(1-p_s)V_{-s}}{1-\delta p_s} \\ &= \frac{(\delta v - c_s)}{1-\delta p_s} + \frac{\delta(1-p_s)}{1-\delta p_s} \times \left[\frac{(\delta v - c_{-s}) + \delta(1-p_{-s})V_s}{1-\delta p_{-s}} \right] \Rightarrow \end{aligned}$$

$$V_s \left[\frac{(1-\delta p_s)(1-\delta p_{-s}) - \delta^2(1-p_s)(1-p_{-s})}{1-\delta p_{-s}} \right] = (\delta v - c_s) + \delta(1-p_s)\left(\frac{\delta v - c_{-s}}{1-\delta p_{-s}} \right) \Rightarrow$$

$$\begin{aligned} V_s &\left[(1-\delta p_s)(1-\delta p_{-s}) - \delta^2(1-p_s)(1-p_{-s}) \right] \\ &= (1-\delta p_{-s})(\delta v - c_s) + \delta(1-p_s)(\delta v - c_{-s}) \end{aligned} \tag{16}$$

Since $\delta \in (0,1)$ and $p_s \in (0,1)$, the term that multiples V_s on left hand side of equation [16] is greater than zero. Thus, the right hand side of equation [16] tells us that the sign of V_s depends on the weighted values from the profits in each state.

Since we have assumed that $c_l < c_h$, then it can be shown that equation [16] implies that $V_l > V_h$.[19] Therefore to ensure ICC for the lender to give loans in all states we

[19] We can write equation [16] as $\kappa V_s = (1-\delta p_{-s})u_s + \delta(1-p_s)u_{-s}$, solving this equation and noting that $u_l > u_h$ we can conclude that $V_l > V_h$.

only need to verify that:

$$\begin{aligned} V_h &\geq 0 \Rightarrow \\ (1-\delta p_l)(\delta v - c_h) + \delta(1-p_h)(\delta v - c_l) &\geq 0 \end{aligned} \tag{17}$$

If equation [17] is not satisfied, then the lender would renege on offering loans if a high cost state ($c_s = c_h$) is observed.

To prove the second statement, note that from equation [1], varying p_h while holding Θ constant requires:

$$p_l = \frac{2\theta_l + p_h\theta_h - 1}{\theta_l}$$

Subbing the right hand side for p_l into equation [17] we have:

$$(\theta_l(1-\delta) + \delta\theta_h - \delta p_h\theta_h)(\delta v - c_h) + (\delta\theta_l - \delta p_h\theta_l)(\delta v - c_l) \geq 0 \tag{18}$$

Following our initial assumptions that $(\delta v - c_h) < 0$ it is straightforward that the left hand side of equation [18] is negative at the limit of $p_h = 1$. Thus, the inequality only holds at low values of p_h if, and only if, the following is satisfied:

$$(\theta_l(1-\delta) + \delta\theta_h)(\delta v - c_h) + (\delta\theta_l)(\delta v - c_l) \geq 0$$

The above condition guarantees that the left hand side of equation [18] that is linear in p_h, is decreasing and is positive at the limit of $p_h = 0$. Thus, it follows that at high values of p_h equation [18] never holds and as a result a rise in p_h holding the stationary distribution constant will make giving loans in all states more difficult. □

Proof. of Lemma 2

The constraints in the lender's problem in equation [15] form a non-empty compact set of feasible (r_l, r_h) pairs, and the linear objective function is continuous. Thus, by the Weierstrass extreme value theorem, an optimal (r_l^*, r_h^*) pair exists. Moreover, since the objective function is (weakly) convex, a local maximum is also the global maximum. Therefor, the problem in equation [15] has a unique solution that takes one of the three following forms:

(1) $r_l = 0$, $r_h > 0$, only one of the constraints in equation [14] binds.
(2) $r_l > 0$, $r_h = 0$, only one of the constraints in equation [14] binds.
(3) $r_l > 0$, $r_h > 0$, both constraints in equation [14] bind.

Direct payoff comparison for the lender between the above three possibilities reveals the optimal repayment rates. The first case imposes that $r_l^{(1)} = 0$, hence the objective function in equation [15] simplifies to $(1-p_l)r_h - c_l$ and equation [14] simplifies to $r_h \leq \frac{v}{1-p_l}$ and $r_h \leq \frac{\delta^2 v(\frac{1-p_h}{1-\delta p_h})}{1-\delta p_l}$. Since the latter inequality implies the former, the maximizer will be $r_h^{(1)} = \frac{\delta^2 v(\frac{1-p_h}{1-\delta p_h})}{1-\delta p_l}$. Thus, the maximum expected repayment will be: $(1-p_l)r_h^{(1)} = \frac{\delta^2 v(1-p_l)(1-p_h)}{(1-\delta p_l)(1-\delta p_h)}$

The second case imposes that $r_h^{(2)} = 0$, hence the objective function in equation [15] simplifies to $p_l r_l - c_l$ and equation [14] simplifies to $r_l \leq \frac{v}{p_l}$ and $r_l \leq \frac{\delta v(1-\delta p_h)}{(1-\delta p_h)-\delta^2(1-p_l)(1-p_h)}$. Since the latter inequality implies the former, the maximizer will be $r_l^{(2)} = \frac{\delta v(1-\delta p_h)}{(1-\delta p_h)-\delta^2(1-p_l)(1-p_h)}$. Thus, the maximum expected repayment will be: $p_l r_l^{(2)} = \frac{\delta v p_l(1-\delta p_h)}{(1-\delta p_h)-\delta^2(1-p_l)(1-p_h)}$

In the third case, both constraints in equation [14] bind. Thus, the optimal repayment rates can be found by solving the system of equation when both inequalities in equation [14] bind. The solution is $r_l^{(3)} = \delta v$ and $r_h^{(3)} = \delta^2 v(\frac{1-p_h}{1-\delta p_h})$. The maximum expected repayment will be: $p_l r_l^{(3)} + (1-p_l)r_h^{(3)} = \frac{\delta v p_l(1-\delta p_h)+\delta^2 v(1-p_l)(1-p_h)}{1-\delta p_h}$, which is higher than the maximum expected repayment in the first and second cases. As a result, $r_l^* = r_l^{(3)}$ and $r_h^* = r_h^{(3)}$.

Finally, in order for the lender's commitment to lend when cost is low to be credible, the low cost should not exceed the expected repayment:

$$\delta v[p_l + \frac{\delta(1-p_l)(1-p_h)}{1-\delta p_h}] \geq c_l \tag{19}$$

□